On Leibniz and *Letters From Leibniz*

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vispo.com
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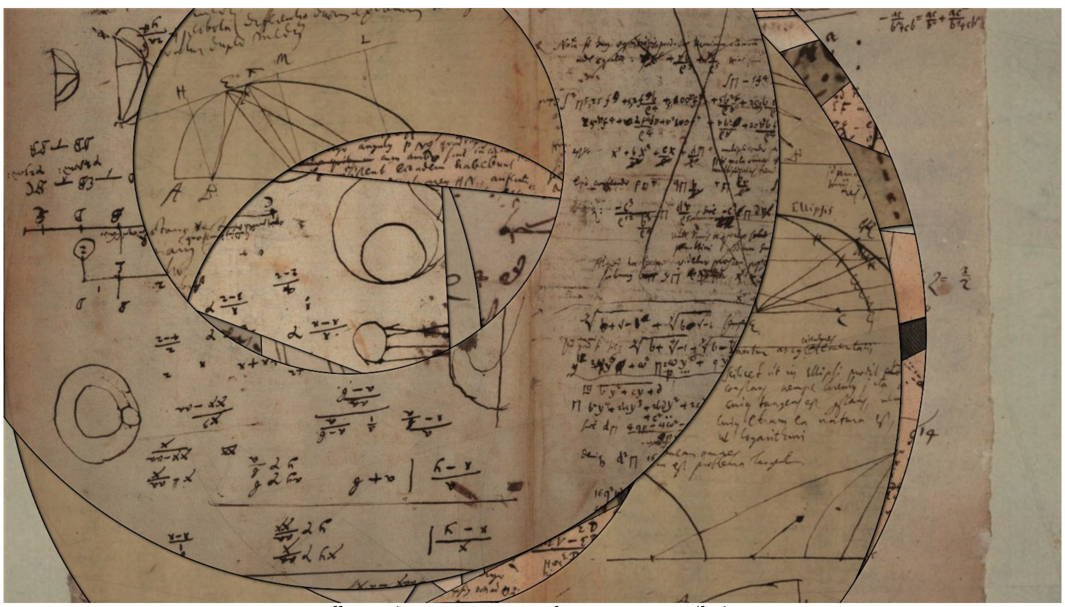


Illustration 1: Image 260 of Letters From Leibniz.

Gottfried Wilhelm Leibniz (1646-1716)

Gottfried Wilhelm Leibniz discovered/created linear algebra and calculus. He was a great mathematician. Relatedly, he thought deeply about computing machines and language. He’s an important figure in the history of the computer, symbolic logic, and the philosophy of computation. He's also one of the great philosophers of his age, with Descartes and Spinoza.

Letters From Leibniz 2.0

This essay is about Leibniz and *Letters From Leibniz* *2.0,* which is an online slideshow of 500 digital collages I made using *Aleph Null*, a graphic synthesizer I wrote in JavaScript+HTML+CSS that randomly samples 238 photos of Leibniz’s hand-written manuscripts, correspondence, and other of Leibniz’s graphical belongings. *Aleph Null* produces a never-exactly-the-same-twice animation that samples Leibniz’s most *visually compelling* writing.

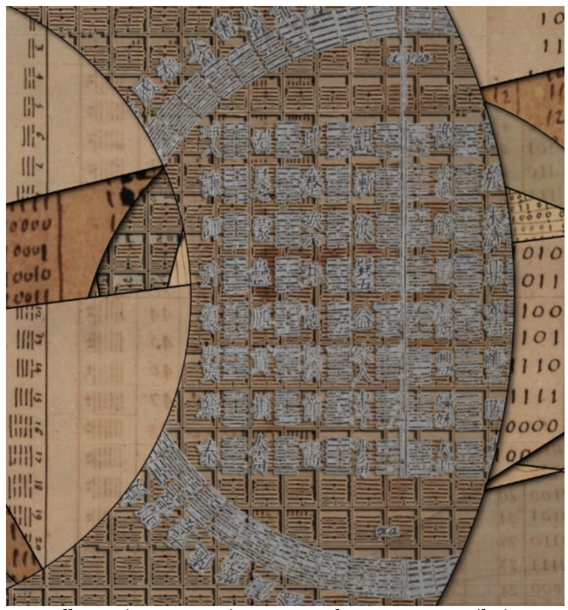


Illustration 2: From image 290 of Letters From Leibniz

Leibniz’s subjects range from his binary number system to other bases, through several Leibniz pages on magic cubes and linear algebra, and considerable on curves, tangents, infinite series and infinitesimals (calculus), to geometry problems, to the design of his computational machine, to meditative visual—sometimes full-page—*enumerations* carried out, perhaps, for later contemplation. 24 of the 238 Leibniz source images deal with binary, ranging from binary arithmetic to the *I Ching*, to hexagrams in base 3, to the decimal expansion of numbers in base 2—infinite series. Leibniz was a poet of the infinite and infinitesimal.

It’s all mixed into a visual show of 500 images, in *Letters From Leibniz*, where you see the source images a bit at a time and put them together, with repeated viewings, on your own. Now you see it. Now you don’t. Later on you see different parts again. You put the pages together, mentally. Then you put the writing together at a deeper level, if you can.

How many times do we typically see each Leibniz source image? Suppose that the 238 source Leibniz images are sampled randomly among the 500 digital collages, and that, on average, we see 6 source images in one digital collage. Then we make 500×6 = 3000 image selections among the 238 random possibilities. So the *expected number* of times we see any particular image is 3000/238 ~ 13. We see each image around 13 times. Some more, some less. That’s enough to usually reveal it all at least once.

All is revealed in the 71 minutes it takes for the slidvid to show all 500 digital collages.

Visual Dimensions of Leibniz's Writing

The visual dimensions of Leibniz's writing are intriguing. Much of his work was never published, during his lifetime, but exists, still, as hand-written/hand drawn manuscripts. It ranges from finished things—that look better hand-drawn than they *ever will* typeset—to hasty arithmetical calculations not meant to be widely seen. Some of these have strong artistic energy to them. Leibniz's work usually has the look of something meant to be looked at. Even when it doesn't, it's often highly expressive. It goes from polished to punk, from exploratory to contemplative, from writing to illustration, illustration to alchemy, alchemy to analysis.

He wrote in Latin, French and German. And he could write in several different fonts or scripts. Also, his mathematical writings are frequently accompanied with rich illustrative diagrams, often in Cartesian coordinate systems.

His fascination with binary (base 2) and the *I Ching* is evident in several pages he created of hexagrams. He also did some work in base 3. Illustration 3 is part of a full-page enumeration of all $3^6 = 729$ hexagrams in base 3. Their resemblance to hexagrams from the *I Ching* is no coincidence, as we shall see.

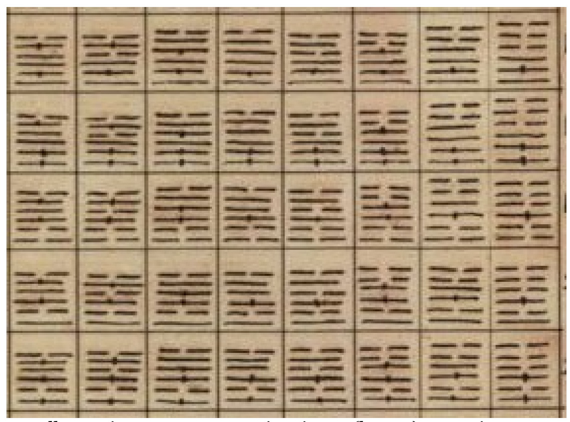


Illustration 3: Hexagrams in trinary (base 3). From image LH 35,3B,7,[2]-1v in the Gottfried Wilhelm Leibniz Library

Leibniz was the first to write about numbers in base 16 (hexadecimal). This is discussed in a 2022 book called *Leibniz on Binary* by Lloyd Strickland. It contains several images from the Gottfried Wilhelm Leibniz Library that are also in *Letters From Leibniz*. Strickland provides English translations of those images, and writes at length about what's going on in each of them.

We see Leibniz's interest in binary also in a clock he devised that has only one hand, displays binary, and is tactile for the blind, or for the sighted when it needs to be read at night in the dark. The clock was never actually made during Leibniz's lifetime.

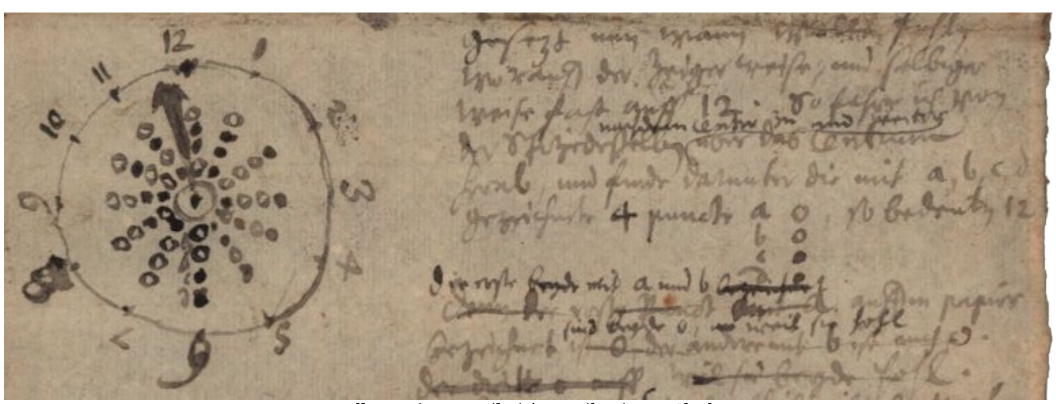


Illustration 4: Leibniz's Tactile Binary Clock
From image LBr. 916, [87] 44r in the Gottfried Wilhelm Leibniz Library

Leibniz also wrote extensively about the calculating machine he created, the *Stepped*

Reckoner, and illustrated those writings at length. In Leibniz's diagrams of the *Stepped Reckoner*, there are lots of interlocking gears. It's useful to think of these in relation to odometers (which are quickly disappearing in favour of digital displays). As we know, odometers have wheels/gears that interlock. There are ten teeth on each wheel of a normal odometer. Cuz odometers are, normally, in base 10. If the number system used is base 4, there will be four teeth on each wheel of the odometer. Thus we see the fundamental relation between gears and numbers in something like a calculator. Gears often represent one digit of a number in an odometer-like construction. Other gears or sub-gears may be for adding, subtracting, multiplying and dividing the numbers.

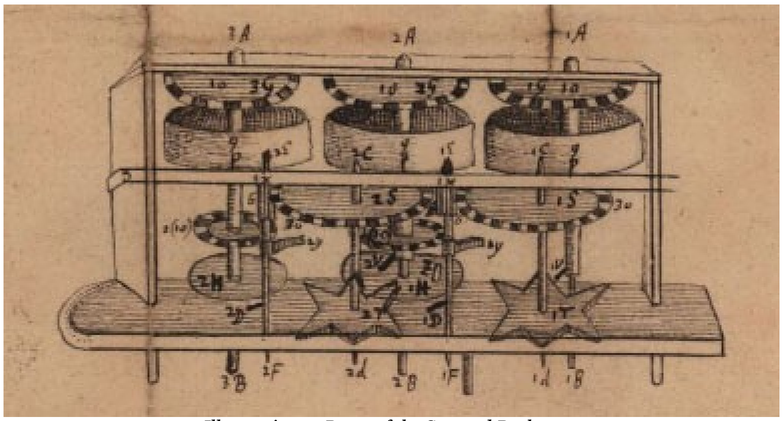


Illustration 5: Parts of the Stepped Reckoner.
From image LH 42,4,1,[17]-9r in the Gottfried Wilhelm Leibniz Library

Other visual dimensions include polynomials not graphed but written in algebraic notations; infinite series of polynomials; his personal calculus notations, including the current integral symbol ∫, and the fractional notation he created for derivatives, i.e. rates of change, in calculus, dy/dx. Also, he drew alchemical/chemical instruments, alchemical symbols, planetary symbols, and zodiac symbols—typically as variables for equations.

Leibniz's motivation for creating calculus was different from Newton's. Newton was fascinated with physics and needed calculus to solve problems of motion. Leibniz was not so involved in physics. He was more interested in the classic dual problems of finding the tangent to a curve and the area under a curve, in contrast to Newton's concern with finding velocities and accelerations etc from physics. And Leibniz was also fascinated with the infinite and the infinitesimal. Mathematically, philosophically and metaphysically. Infinite sequences and series were of great interest to him. I've tried to include lots of images of these issues in his writing.

He also wrote deeply about magic squares and cubes, and illustrated them beautifully with 7 pages of 3D diagrams. These were part of his mathematical investigation of systems of linear equations. He and Descartes are credited as the originators of linear algebra, which is basically matrix math.

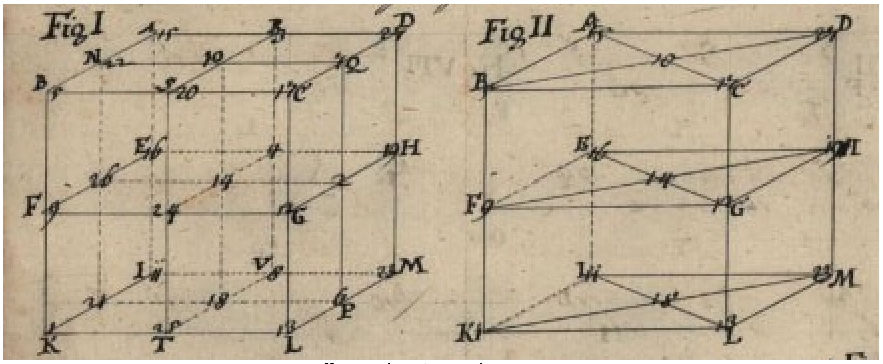


Illustration 6: Magic squares.
From image LH 35,11,5,[15]-8r in the Gottfried Wilhelm Leibniz Library

He also created (or was in the possession of) some interesting concentric, seemingly turnable wheels-within-a-wheel that would have allowed the user possibly to dial into a specific astronomical moment, to see, perhaps, the approximate position of the planets, sun, and/or moon at that moment. Or the level of the tides.

Illustration 7: One of 5 similar constructions.
Image LH,35,15,6,[163]-79br in the Gottfried Wilhelm Leibniz Library

The papers he used were colourful and textural, and have aged to perfection over the course of 300 years. I have mostly left the colour authentic in *Letters From Leibniz*. I changed it slightly, in some cases, for contrast, so that the language is more readable, and the pages, in total, have more variation in colour. All the slides in [Visual Leibniz](), on the other hand, retain the authentic colour.

Leibniz was apparently fond of making page-length enumerations. He did that with hexagrams in base two (binary) and base 3. He also enumerated all binary numbers of up to 16 digits long. His enumerations seem to have been for meditation. Perhaps to see patterns in the enumeration, or graphical enumeration as a way to pass the time, like counting sheep. Hard to say why, but he did like to enumerate combinatorial things.

Visual Leibniz

Visual Leibniz is a slideshow of the unaltered source images in *Letters From Leibniz*. It shows you the 238 images of Leibniz's work that *Aleph Null* uses to produce the digital collages. *Visual Leibniz* contains parts of Leibniz's hand-written, mathematical manuscripts, correspondence, 'technica', 'militaria', 'Leibniz-Handschriften Aufzeichnungen zur Rechenmaschine LH 42, 4, 1', and other astounding projects of Leibniz, genius for hire.

Letters From Leibniz is a visual reading of Leibniz's visual writing—focusing on the graphical nature of his well-crafted manuscripts. It's a study of Leibniz's work. Looking at *Letters From Leibniz* is interesting as an experience of art—and of Leibniz's work.

Whereas looking at *Visual Leibniz* is an experience of Leibniz's unaltered work.

When you use AI image-generators, you almost never know what images were used in the AI's training, or who made the images, or anything at all, explicitly. By contrast, in *Letters From Leibniz*, we see the source images in their entirety, in a separate slideshow. And the Gottfried Wilhelm Leibniz Library is onboard with the project. They say that the images are correctly cited and everything complies with their policies.

The art and the source images are both part of the project. That isn't true of text-to-image AIs. Wouldn't we like to be able to see the image set an AI has been trained on, or query it on its source images and artists? Don't the AI-makers owe it to the people who made the images on which their programs are trained? I think AIs should be queriable concerning their training source material. In this fascist, Trumpy era of shady deals and downright criminality, the way that the makers of AIs treat artists and institutions is typically deplorable.

To read *Letters From Leibniz* with depth, you also read *Visual Leibniz*. Though it's possible to enjoy *Letters From Leibniz* simply as an art project.

Leibniz's writing is beautiful. He is his own graphics department. Mathematicians tend to draw a lot of diagrams. And produce tables of equations, or other graphical symbolic structures. Leibniz is known for not only the depth of his work, but its breadth. He pursued and wrote about a wide range of fields, many (but not all) of which profit via illustration in visuals, within the writing. He was a master illustrator of his own work.

Letters From Leibniz would not have been possible without the generosity of the Gottfried Wilhelm Leibniz Library. They permit use of the 200,000 Leibniz pages they have in their archive, as long as the library is properly credited and sited.

If Leibniz were to have written about 11 pages per day—every day—for 50 years, that would result in 200,000 Leibniz pages. The man wrote a lot. A lot. One of the things that means is this: he had the opportunity to be an artist of his type of writing. It seems he took that seriously. Even in his hand-written mathematical manuscripts. Visually, the images in *Visual Leibniz* are remarkable.

You can puzzle over each page at length. And by *at length*, I mean you're *never ever* going to get it. Or, instead, you can *get* many of these pages in terms of a few subjects Leibniz wrote about:

- Characteristica Universalis (universal language)
- Computing machine (the mechanization of reason)

- Binary arithmetic, hexagrams and numbers in other bases
- Calculus
- Geometry
- Linear algebra and magic squares/cubes
- Architecture
- Combinatoria
- Technology (other than his machine)

Computing, the characteristica universalis, and binary

The calculating machine he created, the *Stepped Reckoner* or *Leibniz calculator*, was the first one capable of addition, subtraction, multiplication and division. He tweaked this machine for decades; several versions were created. Apparently some of the important demos did not go well, and there was at least one problem that kept it from reliably calculating correctly. And it was hard to crank when multiple registers had carry-overs, like when an odometer goes from 99999 to 100000. It was a money-pit he sank large amounts of money into, over decades—and it was never really finished or quite right.

Like Leibniz's dispute with Newton over credit for the invention of calculus, the *Stepped Reckoner* made Leibniz the Rodney Dangerfield of the upper stratosphere of intellectual achievement. I think of him as the William Shakespeare of the Combinatorial Art. He was very optimistic, but that wasn't because he was a perpetual winner. His optimism was religious and philosophical. He was a working man who did much of his real work in his spare time. He had the infinite in him.

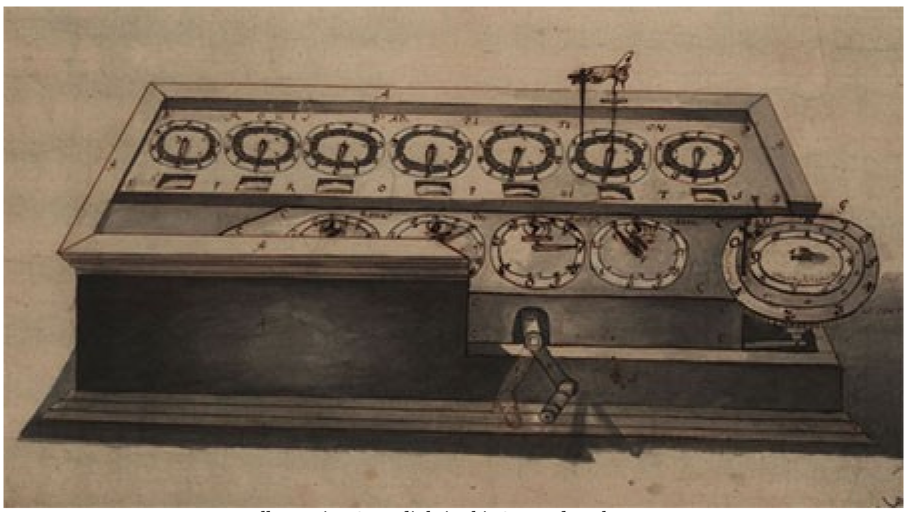


Illustration 8: 12 dials in this Stepped Reckoner.
Image LH 42,5,[61]-31r in the Gottfried Wilhelm Leibniz Library

Leibniz also drafted—or had created—several graphics in this project. One is of hexagrams of the *I Ching* arranged in a circle. He could have drawn the hexagrams, but probably not the Chinese. Leibniz was fascinated with the *I Ching* as a binary, combinatorial engine. He felt that binary would be good for expressing things with machines. He was also impressed with the way that the *I Ching* deals with all possible human situations.

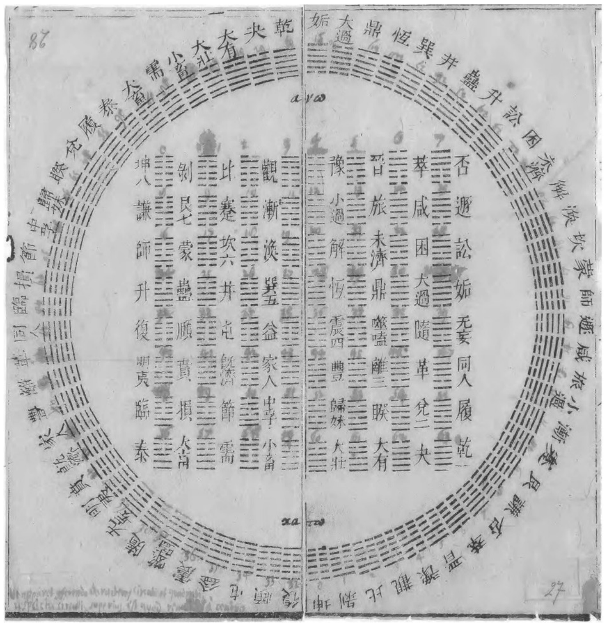


Illustration 9: Leibniz's I Ching graphic

Leibniz valued binary for several reasons. It's an alternate number system to base 10 and provides fresh light on some problems, such as the infinite series

$$\begin{aligned}
&\ \ 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \ldots \\
&= 1/2^1 + 1/2^2 + 1/2^3 + 1/2^4 + 1/2^5 + \ldots \\
&= 0.11111\ldots \text{ (in base 2)} \\
&= 1
\end{aligned}$$

Another way to see the above infinite series is as the decimal expansion of the binary number 0.11111... which converges to 1.

Similarly, you can look at base 10 numbers as infinite series such as:

$$\begin{aligned}
&\ \ 9/10 + 9/100 + 9/1000 + 9/10000 + \ldots \\
&= 9/10^1 + 9/10^2 + 9/10^3 + 9/10^4 + \ldots \\
&= 0.9999\ldots \\
&= 1
\end{aligned}$$

The infinite series equals 1 if you believe that it converges to 1.

Binary numbers and their decimal expansions gives us a new perspective on some infinite series: they're just the decimal expansion of numbers in base 2.

Leibniz also valued binary as possibly useful in his dream of a universal language. He also thought binary might be useful in machines. He had no idea how right he would be in 300 years.

Leibniz also loved the *I Ching* for its connection with binary arithmetic. Hexagrams can be thought of as base-2 numbers cuz *I Ching* hexagrams are made up of only two symbols. Leibniz is known as a sinologist. He was convinced that the creators of the *I Ching* were aware of more binary arithmetic than they probably were.

The *I Ching* is similar to the machine that Leibniz dreamed of in that, just as Leibniz thought that statesmen would consult the machine to generate the answers to the problems of the world, the *I Ching* is consulted as a kind of oracle for answers on all manner of questions.

The *I Ching* is associated with Chinese philosophy. With Taoism, in particular. Lao Tse, the most famous of the Taoists, is said to have been one of the initial inventors and writers of the *I Ching*. I expect that was Leibniz's view, in any case.

Perhaps to Leibniz the *I Ching* was an amazing device of 'divination' in which binary was important as a structuring principle, as an important part of that device and its associated dualistic, generative philosophy. I think he viewed the *I Ching* as an indication that his dream was shared, in important ways, by the ancient Chinese.

Laplace, the great French mathematician who lived two generations after Leibniz, said:

> "Leibnitz saw in his binary arithmetic the image of Creation … He imagined that Unity represented God, and Zero the void; that the Supreme Being drew all beings from the void, just as unity and zero express all numbers in his system of numeration."

Tobias Dantzig, in his amazing book *Number* says:

> It is the mystic elegance of the binary system that made Leibnitz exclaim: Omnibus ex nihil ducendis sufficit unum. (One suffices to derive all out of nothing.)

There's another diagram he created earlier—for his first book—which was titled *De Arte Combinatoria* (On the Combinatorial Art), his dissertation when he graduated from school, which he tweaked to turn into the book. We're told that the diagram involves the "characteristica universalis", perhaps in the sense that each universal character, or symbol, was to have properties generated from the use of the combinatorial engine depicted in the below diagram.

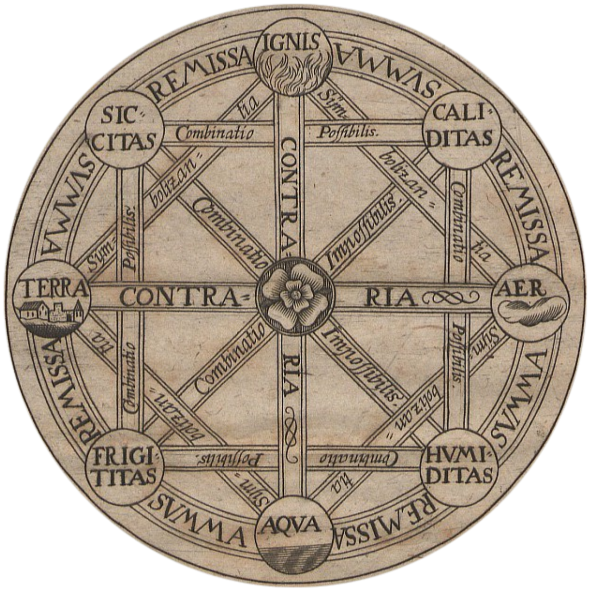


Illustration 10: Frontispiece of Dissertatio de arte combinatoria, 1690

For instance, each thing in the world would be made of the four elements, as was commonly thought at that time (and earlier): earth, air, fire and water or, in Latin, terra, aer, ignis et aqua, which we see in the above diagram. So that each universal symbol would have numbers or other measures concerning quantity and possibly structure for each of the four elements.

We see, in the above graphic, "SICCITAS", "CALIDITAS", "FRIGITITAS" and "HUMIDITAS".

SICCITAS = Dry
CALIDITAS = Heat
FRIGITITAS = Cold
HUMIDITAS = Humid

Just as each universal character/symbol was supposed to have its elemental description in terms of earth, air, fire and water, so too would each universal character/symbol have a description in terms of hot, cold, dry and humid. This is terminology associated with the humors, i.e., the Galenic medicine of that time. The four elements and four humors were used, in various proportions, orders and whatnot, to describe healthy or diseased states of the body.

The idea, then, of the above diagram is more or less evident. Just as the elements and humors are used to describe different states or conditions of the body, to define temperaments, so too can they be used in a process that tags each idea and thing in the entire world with a unique combination of elements and humors.

The above diagram has other Latin in it. "Combinatio Possibilis" means the two joined nodes can be combined. Whereas "Combinatio Impossibilis" means the two joined nodes cannot be combined. There is more language and structure in the diagram that describe what can be combined.

Leibniz thought of the diagram as describing a machine that generates combinations of elements and their properties. You can also think of it as enumerating each of the universal symbols it describes, much the same way that interconnected wheels (odometers) can enumerate all the numbers expressible with the digits on the wheels. The machine does not have enough permutations to name everything in the world unless each thing is named via a sequence of such settings, or something like that. In any case, the diagram is more like a hint—not even a 'proof of concept', rather than a fully working thing.

Obviously the system has quite a long ways to go before it could begin to do what Leibniz wanted it to do—which was soaringly ambitious, beyond even today's capabilities—but we see a bit about how the language Leibniz envisioned for the "characteristica universalis" is also involved in machines, in Leibniz's way of thinking. He thinks of computers as being combinatorial devices, like multi-purpose odometers. This is enough room, he thinks, to mechanize reason fully and completely. It was a fascinating and deep dive into matters that weren't clarified until the 20th century.

Leibniz's 'big picture' was prophetic—not so much via divine inspiration—though who can say for sure?—but, instead, by a philosophy in which a "universal science", or "mathesis universalis"—basically, what we now call a formal system—could be devised/discovered. A language based on a dictionary of universal symbols (or *characteristica universalis*) would enumerate *every* idea and thing in the world. And a "calculus ratiocinator", or calculating machine, would generate truths—and only truths—about propositions expressed in universal symbols.

We could also call the *calculus ratiocinator* a theorem-proving program. It would be capable of proving any truth expressible in the language. Or so Leibniz thought, of course—it's a reasonable idea he named the principle of sufficient reason. However, in 1933, Kurt Gödel showed that, in any sufficiently powerful formal system, there are always going to be unprovable truths. Not every truth is provable. Some things must be accepted as true without there being a reason for why they are true. There are always going to be more unprovable truths, also. Cuz every sufficiently powerful formal system has them—especially the one you thought you just added the last axiom to.

The idea of the formal system , or axiom system, is important in the historical development of computers. Why? Going back to the Greek formalization of geometry as proceeding upon the axiomatic method, we have a long and fruitful history in mathematics itself of creating systems that specify rules of inference about propositions, i.e., how to validly construct propositions, reason about the propositions, and validly draw conclusions from propositions. The universe of discourse in such systems is, typically, limited to geometrical entities: sets, subsets, points, lines and whatnot. The propositions are about those sorts of things.

But, from the time even before the Greeks, to now, what we would like is something that can tell us truth about things *more complex* than sets, subsets, points, lines and whatnot. We're usually concerned with things that don't at all seem obviously amenable to any good treatment whatsoever in terms of sets. There is continual need for systems that go beyond the systems we currently have. But they can only advance so fast.

Leibniz thought that if he had five years away from his day job, five years to create a reasoning machine, he could do it—and it would change the world unlike anything ever seen before. He expressed regret, near the end of his life, for not having gotten closer than he did to realization of his dream of a reasoning machine.

We can see now that even if he had considerably more than 5 years, it would have been very difficult for him to have produced anything remotely as useful and general as the modern computer.

Because the history of formal systems, from Leibniz to Turing, necessarily involves the work of people such as George Boole (Boolean logic), Gottfried Frege (symbolic logic), Georg Cantor (set theory), David Hilbert (meta mathematics), Kurt Godel (incompleteness), and Alan Turing (the universal computer), most of whom are immortals of math and logic. They hadn't been born yet. But there is a line of progress and development toward the computer from Leibniz to Boole, to Frege, Cantor, Hilbert, Godel, and to Turing. Seven generations, from Leibniz to Turing, over the period 1666-1936, or 270 years. That's 13 generations. So the vision skipped some generations, or we don't know the full story. Some trace it back further to Llull in the 14th century.

Going from systems that support reasoning only about things like points and lines to reasoning about things in the world is a larger step than it appeared to be to Leibniz, and required from 1666 (Leibniz publishes *De Arte Combinatoria*) to 1936 (Turing's inauguration of the theory of computation with the publication of his earth-shaking paper on the *Entscheidungsproblem*), a period of 270 years, to bring Leibniz's dream of a machine capable (in theory) of reasoning into the world—though, even now, we seem some distance from artificial general intelligence (AGI). AGI can understand or learn any intellectual task that a human can. It can also understand symbol systems, belief systems, and the meaning behind what it's doing. If it existed, which it doesn't. Yet.

Although Leibniz was a prophetic genius, he wasn't the only one who understood that the process of reasoning could, in theory, be mechanized. Descartes, for instance, was on to that. Indeed, we need to view the history of mathematics/logic as never entirely divorced from concern with systems of reasoning beyond simple geometrical entities—toward helping us with other contemporary problems we face. In the Handbook of the History of Logic, Volker Peckhaus says:

> "...projects for the development of artificial languages were common in 17th century intellectual circles. They were pursued for their expected benefits in promoting religious and political understanding, as well as commercial exchange."

There is an ongoing, pressing need for things that help us toward "religious and political understanding"—and making a buck. But, more generally, machines help us with many problems. Not a lot about sets and subsets, points and lines. The point is that *our problems* continually focus attention on the need for systems that are capable of helping us reason about *anything*, help us understand anything, not just abstract sets. This is why the dream of the general reasoning machine is so old and venerable in its association with the greatest mathematical/logic progresses/discoveries of previous ages.

The problem is not simply to make a machine to solve our problems for us, like a magic, wise genie. It's to make a machine that allows us to understand and form better decisions ourselves. Whether that's through statistically predictive Large Language Model AIs or what was imaginable in Leibniz's age.

What would Leibniz think of today's AIs? Ironically, they are not very good at math or logic/reason. They are simply predictive of the next character/word/phrase/sentence/paragraph, based on their training involving millions of documents. And Leibniz was all about tools that really helped humanity. I think he would say that his dream has not yet been realized.

Letters from the other side

One of the *Aleph Null* brushes used in the below images is the “Leibniz Shards” brush. A number of points along the perimeter of the canvas is specified, via the *size* slider. The "Leibniz Shards” brush then proceeds to join those random perimeter points by straight lines, going from one point to another, eventually visiting them all, until finally returning to the point it started at, and thus closing the figure. The resulting closed figure is then filled, using the "even-odd rule"—that fills the way I fill when I doodle. It's filled with an image of Leibniz's work.

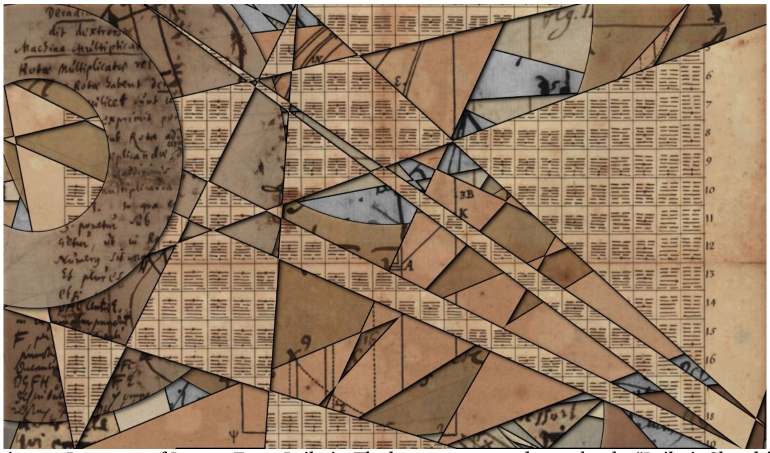


Illustration 11: Image 54 of Letters From Leibniz. The hexagrams are drawn by the “Leibniz Shards” brush.

The images are letters from me to you, also. I create art that involves visual writing, usually. I think of these images as a kind of writing. As a kind of poetry, even. But *Letters from Leibniz* is also my attempt to get Leibniz to write me new letters, and to work with this inspiring genius as a means of reading him, appreciating him, looking at his work not solely as math and philosophy but also visual art and a really startling type of writing. I wonder how startling people of the 17th century found his writing? I suspect they too would marvel at it not only as thought but as something to look at very slowly. Writing such as his is a marvelous thing to behold.

The pieces form a new kind of ‘page’ of digitally collaged writing as image. A related piece of work I did was in collaboration with the Russian/Israeli poet Kirill Azernyy. In our piece, *End of Recogniton*, I used *Aleph Null* (and Photoshop) to digitally collage a story Kirill wrote. The result is a different sort of approach to fiction.

Another related work I made is *City on the Other Side of Time*. This is a slidvid of 500 curated images, as is *Letters From Leibniz*. The source images are a subset of what I call “Alchemical Cosmography”.

Leibniz's Vision

My fascination with Leibniz began when I studied calculus, in my late teens—and learned that he discovered/created it independent of Newton. I didn't know anything about his contribution to computing or philosophy, at that point. But I loved studying calculus. The idea of the limit of an infinite sequence, or the limit of a function as x approaches a particular value, captures something about inevitability, the inevitable.

Leibniz's was the driving vision about the significance of computing to human affairs during his time and the time leading up to Godel and Turing in the 20th century. Leibniz was the main prophet of the computer.

He had a strong vision of computing, and thought of its promise in the most optimistic and powerful terms. He thought that computing machines of such moral value could be devised that statesmen, when they differed in opinion on a matter—even ethical, philosophical matters—would say "Let us calculate!" and would find the right answer with the machine.

This is the opposite of Weizenbaum's (he wrote Eliza) position. He said there are certain things no computer should be tasked with. Namely things requiring wisdom.

In the beginning of thought on important matters, such as computing, we see naive philosophical claims. As time goes by and we understand more about what is possible and what isn't, we come to a more nuanced position. We do not see statesmen saying "Let us calculate" to solve their differences on questions of ethics or other matters. Though they probably do bring their economic forecasts and impact models, etc, that computers have been indispensable in generating.

But, even so, many believe that there will come a day when AI are sentient and even immensely wise, and will be indispensable even in ethical, philosophical questions. Why do they believe it? Because it's theoretically possible. Though, at the moment, the bots I encounter are neither sentient nor even remotely wise. Yet they are vastly superior to bots from 10 years ago.

Leibniz was passionately convinced that reasoning could be fully mechanized. He wanted to create a language in which each thing in the world and in the world of ideas had a word for it, a "characteristica universalis", and a machine that could use that language to mechanize reason.

Naive? Yes, of course. Little was known about such things. But, even so, we see in his vision the beginning of a quest to mechanize reason that continues in our day.

Martin Davis, himself a renowned logician, wrote a fabulous book called *The Universal Computer*, in which he looks (chronologically) at the life and work of seven logicians/mathematicians, from Leibniz to Turing, whose work was crucial in the eventual theory of computation that we have today which is indispensable in the elegant conceptualization of computing machines.

Davis also traces the development of the computer through the 'crisis of foundations' in mathematics/logic. The 'crisis of foundations' is, in itself, a fascinating excursion in the history of ideas that has not only strong connection with the development of boolean logic, set theory, and symbolic logic, but also fascinating philosophical significance in matters of epistemology, i.e., how we verify what we think we know.

Since Leibniz, it's been theorized that one way to be sure of what we know is to mechanize the logic we use to prove what we think we know. In other words, if we could make a machine where each step of the argument was purely mechanical, there would be little or no room for human error. It's kind of slippery, of course, even then. What's to stop a mechanical step from containing a design flaw? But if we can get it to the point where each mechanical step is sound, each inference is solid, then we could at least say that the machine reasoned correctly according to the rules we built into its mechanisms.

Because there is so much business investment in computers, we tend to think that *business* drives the development of the technology. Perhaps it does, now. But from the time of Leibniz to Turing, it was *philosophy* that drove it: the quest to create foundations of math/logic that we could be confident about. The quest for solid knowledge.

Alan Turing created the modern computer not to usher in the age of computing, but as an important step in proving that there are some tasks that no machine will ever accomplish. He invented the modern computer to solve a math/philosophy problem.

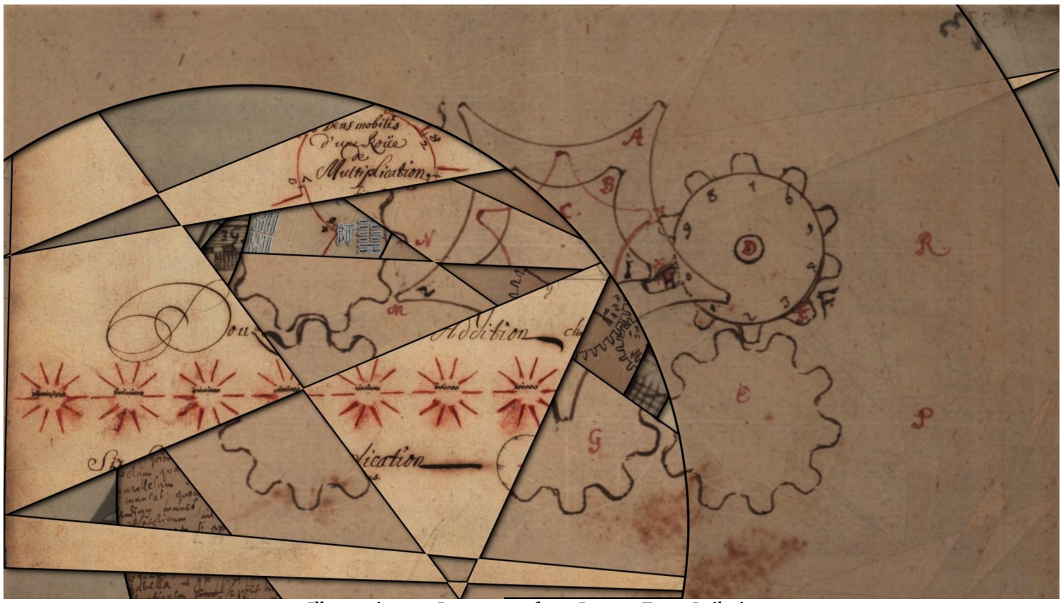


Illustration 12: Image 490 from Letters From Leibniz.

Cultural inheritance

As a computer artist, this history is an important part of my artistic inheritance, my intellectual culture. Computer art inherits the soaring philosophical aspiration, the quest for knowledge and knowledge about knowledge, inherits this long search for understanding, for machines that can help us not only calculate mathematical things but help us in matters as deep as ethics and governance. And last but not least, machines that we can make new and amazing art with.

When I was an undergraduate, I did a degree in English and Math. One of the things I loved in math was the work of Georg Cantor. He’s a central figure in this story, as he explored set theory deeply—and even proved some things worth knowing about infinity. I always felt that work was the most beautiful math I'd read and understood. Now I feel that it is not only part of my inheritance as a mathematician, but as a computer artist. The creation of that sort of beauty, that sort of understanding, is not unrelated to what we seek to create as computer artists. And Cantor is one of the seven logicians Martin Davis writes about in his fabulous book *The Universal Computer*.

As computer artists, part of our cultural inheritance is the history of the development of the computer from Leibniz to Turing and beyond. That history, it turns out, is not only the history of the computer, but the history of some of the most prominent philosophy and mathematics/logic from Leibniz to Turing and beyond.

Leibniz and Linear Algebra

A quick web search of “linear algebra” reveals that the first names associated with it are Leibniz, for his work on determinants, and Descartes. Don’t worry if you don’t know what *determinants* are. The point is that linear algebra is the math of matrices, and Leibniz was one of its creators. He created calculus *and* linear algebra. These were the two fields studied in first year mathematics at university when I went to school 300 years after Leibniz.

Everybody knows what a matrix is—because of the film *The Matrix*. A matrix is a grid of numbers. They’re important in representations of bitmap images—and of the world more generally—in computers. But what was Leibniz’s interest in them?

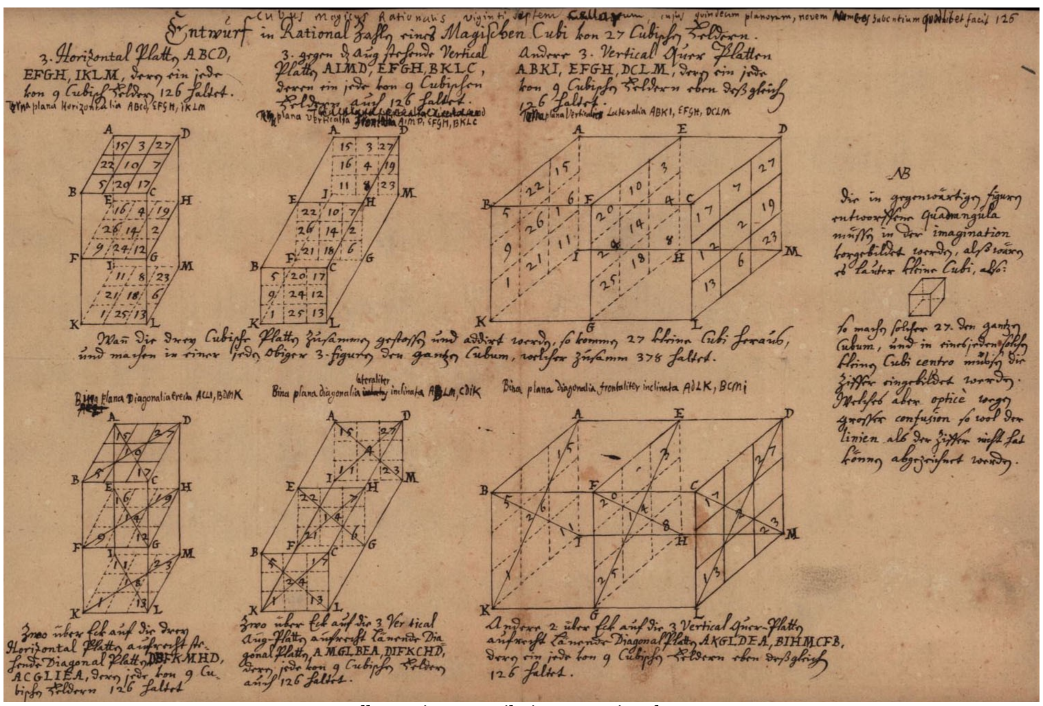


Illustration 13: Leibnizean magic cubes.
Image LH 35, 11, 5, [1] 1r from the Gottfried Wilhelm Leibniz Library.

Throughout his life, he was devoted to the mechanization of logic, of reason. So that we could solve the problems of the world with the use of an aid to reason: a machine or *ratiocinator* that, with the use of a universal language, a *characteristica universalis*, could make sense of and reason perfectly about any situation under the sun. Perhaps he understood that matrices would be important in that whole venture. They are. The grid is fundamental to mathematical representations of spatial things in coordinate systems. Matrices can also be important to non-spatial things; they aren’t necessarily spatial. A coordinate system is always already a grid (or a deformed grid).

An example of how matrix math is useful is in how they can be used to find answers to questions about magic squares and cubes. Leibniz did some intriguing work with magic squares and cubes. There are at least 7 Leibniz pages/images from the Leibniz Library in

Hanover that contain wonderfully visual magic cubes drafted by Leibniz. They are images [54-60](54-60) in *Visual Leibniz*.

To focus, for the span of 14 digital collages, on linear algebra in *Letters From Leibniz*, I created a subset of the 238 Leibniz source images that deal with the subject. I also found some images in the Hanover library of Leibniz doing determinant calculations and related work with linear equations, and put 4 of those into the mix too.

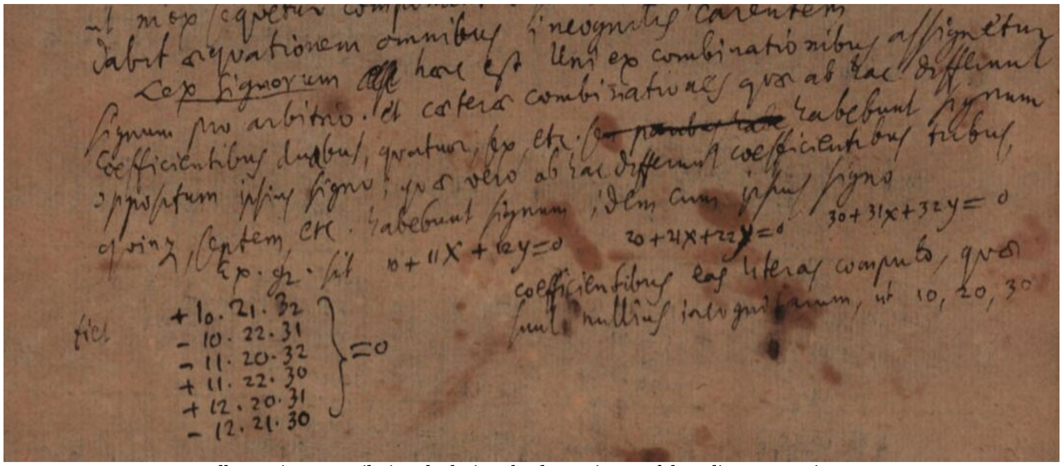


Illustration 14: Leibniz calculating the determinant of three linear equations.
From image LH 35, 3 A 37, [1] – 1r from the Gottfried Wilhelm Leibniz Library.

Then I created 14 digital collages that sample the 11 linear algebra images, and only the 11. They are images 463-476 in *Letters From Leibniz*.

Illustration 13 has ‘magic cubes’ on it. Each cube has three of the 9 squares emphasized. They depart from the usual form of the magic square, where the row, column, and diagonal sums are the same. Here's an example of a canonical magic square. The row, column and diagonal sums are 15:

4 9 2
3 5 7
8 1 6

By the way, we might ask what's magical about a magic square. Wikipedia tells us

> "Magic squares have a long history, dating back to at least 190 BCE in China. At various times they have acquired occult or mythical significance, and have appeared as symbols in works of art. "

There’s an altar designed by Alistair Crowley that includes two magic cubes of a different kind, but seems to at least allude to work with mathematical magic cubes.

In the case of the Leibniz 'magic cube', in illustration 13, we see the *same* cube in all six drawings, but from different perspectives, in the top three cubes. The bottom three cubes are concerned with the sum of the diagonals of that same cube—which are always 42.

As we shall see, the cube is ‘magic’ from the three perspectives on the cube in the top row in illustration 13.

15	3	27
22	10	7
5	20	17
16	4	19
26	14	2
9	24	12
11	8	23
21	18	6
1	25	13

A cube is made up of three squares, and each square is 3×3. To the left is the cube at top left in illustration 13. Let's call the three squares the *z-squares* cuz they're stacked along the *z* axis of the top left cube in illustration 13.

Looking at any of the these squares, we see that the diagonal sum is always 42. And the total sum of each square is 126 = 33+42+51 = 39+42+45. Also, the cube uses the numbers from 1-27 with no repetitions.

Row sums: 39, 42, 45
Column sums: 33, 42, 51

15	3	27
16	4	19
11	8	23
22	10	7
26	14	2
21	18	6
5	20	17
9	24	12
1	25	13

If we redraw the same cube so that we see the one at *top middle* of illustration 13, we get the three squares to the left. Let's call them the *y-squares* cuz they're stacked along the *y* axis in the cube in illustration 13.

Again, the total sum of each square is 126 = 39+42+45 = 15+42+69. Also, the diagonal sum, again, is always 42. Also, again, the cube uses the numbers from 1-27 with no repetitions.

Row sums: 39, 42, 45
Column sums: 15, 42, 69

5	22	15
9	26	16
1	21	11
20	10	3
24	14	4
25	18	8
17	7	27
12	2	19
13	6	23

If we redraw the same cube so that we see the one at *top right* of illustration 13, we get the following squares. Let's call them the *x-squares* cuz they're stacked along the *x* axis.

The diagonal sum, again, is always 42. Also, again, the cube uses the numbers from 1-27 with no repetitions. And, again, the total sum of each square is 126 = 33+42+51 = 39+42+45 = 15+42+69.

Row sums: 33, 42, 51
Column sums: 15, 42, 69

Leibniz presents multiple views of the same thing, the same cube. It's *cubistic* in that sense. It's also *cubistic* cuz it has a lot of cubes. He'd never heard of *cubism,* though. That was 250 years later in art. But some of the same things apply:

- *Artistic cubism* involves presenting multiple perspectives simultaneously. Check: in illustration 13, we see six different perspectives on the same grid of numbers.
- *Artistic cubism* is painterly. Well, no, Leibniz's magic cubes are not painterly. *Cubism* generalizes beyond the painterly. It's the multi-perspectival that's crucial. Visually and conceptually.
- *Artistic cubism* intertwines the different perspectives beautifully. Check. That's the moral of the above demonstration. The magic cube is magic however you slice it. It's integrated in all three directions of 3-space—in the sense that the numbers are 1-27 with no repetitions; the sums of the diagonals are always 42; and the

sum of any square is 126. This is all very integrated in all directions. A strong, stable structure. The 'magic' is in how it manages to satisfy more related constraints than you might think were possible. It's balanced and harmonious at every turn, yet it has variation.

Linear algebra is a powerful approach to things like magic squares and cubes that helps solve the riddle of how many solutions there are that satisfy a given set of constraints, and how to generate each and every one of those solutions. It takes a bit of the 'magic' out of 'magic squares and cubes', cuz it answers many of the questions about them. But, like any powerful theory, it has its own beauty. And wide utility.

Binary and Machine-Oriented Digital Collages

80 of the 500 digital collages sample 24 Leibniz images concerning his binary number system and other bases. These images are salted through *Letters From Leibniz*. This was to focus on the binary a bit—well, more than a bit. Binary and the arithmetic of other *bases* was widely taught in schools for a long time. Perhaps it still is. Lots of people remember it and loved it—more than the ones who studied and enjoyed calculus. Leibniz did a lot of work in binary and other bases, as we can see in his mathematical manuscripts.

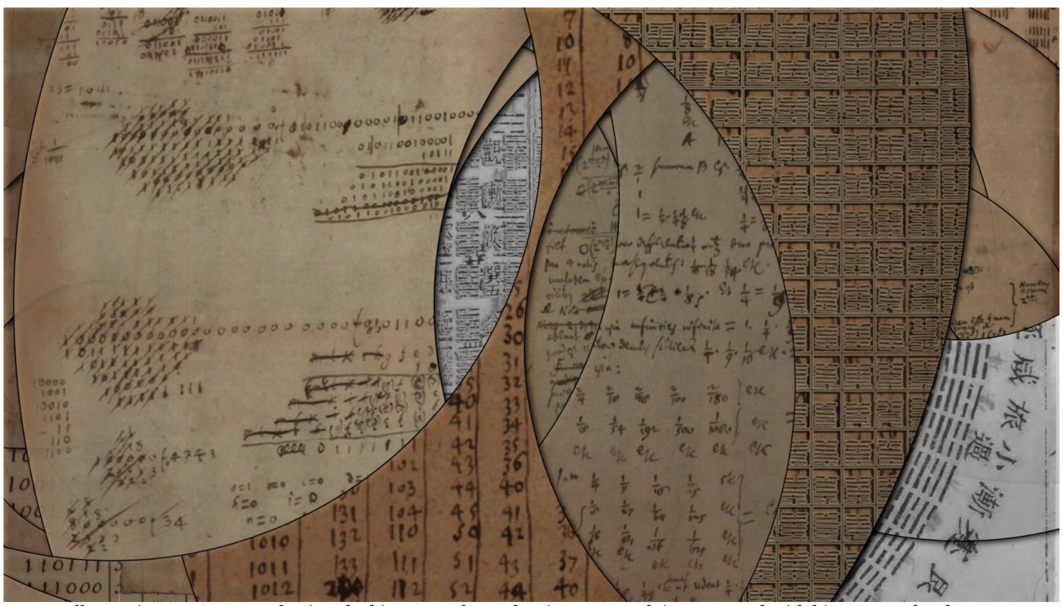


Illustration 15: Generated using the binary+ subset of 24 images. Each is concerned with binary or other bases.

I made 22 images (478-500) that sample 15 machine-oriented Leibniz images. They use images of gears, the *Stepped Reckoner* (his calculating machine), and other machine-oriented images. These are at the very end of *Letters From Leibniz*. They're significant to me because Leibniz is significant in the history of computing—the history of computing is significant to me as a computer artist, math guy and citizen of the 21st century. Just as the history of painting is of interest to a painter.

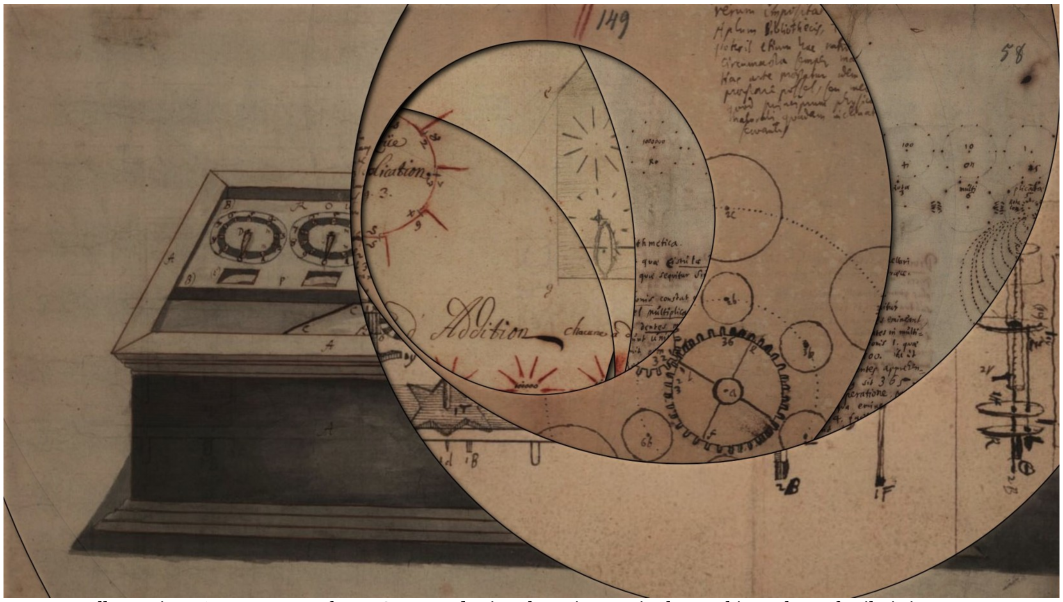


Illustration 16: Image 500 of 500. Generated using the 15 images in the machine subset of Leibniz images.

I selected many images based on whether they involve calculus. The more, the better, was my criterion. They are salted through the 500 images of *Letters From Leibniz*. But I didn't put together a sequence of calculus images, unlike binary+, linear algebra and machines, because I'm more interested in these subjects than in calculus, in this project, cuz I'm a computer artist interested in Leibniz's place in the history of the computer and computer art.

Visual Riddles

Letters From Leibniz poses visual riddles by obscuring parts of the writing. There are a few ways to resolve the riddles.

One is by watching more of *Letters From Leibniz*. All is eventually revealed.

Another is to look at the image of interest in *Visual Leibniz*. That obscures nothing.

A third way is to fill in the blanks oneself, the preferred way to do it—the deep reading approach. That's how Leibniz would do it. That's how Leibniz had to do it.

Letters From Leibniz 2.0 vs 1.0

Version 1.0 of Letters From Leibniz only used about 100 Leibniz source images. Version 2.0 uses 238. So 2.0 has a lot more of Leibniz's work in it than version 1.0.

Version 1.0 used some images that weren't Leibniz's work. Version 2.0 uses *only* images known to have been Leibniz's, which he either wrote or had in his possession and contained his work, such as the drawings of the *Stepped Reckoner*. Perhaps he drew them. I'm not sure. The drawing is of his calculating machine, in any case.

Version 1.0 is a slidvid of 158 images. Version 2.0 is a slidvid of 500 images.

The essay in version 1.0, which is in the slidvid itself, is an earlier version of the essay in version 2.0, which is in a PDF.

Version 1.0 does not contain a separate slideshow of the source images. Version 2.0 contains *Visual Leibniz*, a slidvid of the 238 source images.

The black window in version 2.0, if you scroll down, contains thumbnails and signatures of all 236 source images from the Gottfried Wilhelm Leibniz Library. They're clickable. You view the larger image in a new browser tab if you click a thumb.

In version 2.0 you can get to the source images two ways. You can get to them 0 ways in version 1.0. In 2.0, you can get to them either from the black window of *Letters From Leibniz* or from the separate slidvid called *Visual Leibniz*.

Version 1.0 is not in compliance with the Gottfried Wilhelm Leibniz Library's policies on image use. Which is why, if you visit it, it redirects to version 2.0, which *is* in compliance with the good library's policies.

I wanted version 2.0 to be in compliance with the library's policies cuz, after endless corporate AIs being trained on millions of images and billions of texts, not acknowledging any of them, it seemed like time for a change.

Letters From Leibniz 2.0 explores the relationship that can exist between artists and an online image archive. Rather than a relationship of exploitation, it's mutually beneficial. They help me in allowing me to use these marvelous images. I help them in acknowledging their marvelous work and loving their vision of making Leibniz's work available around the world with only minimal requirements for crediting.

I am reading a book by Lloyd Strickland called *Binary in Leibniz*. Images from the library in Hanover are crucial to this book. The structure of the entire book revolves around looking at a few images from the Hanover archive.

That's the sort of effect this archive has on Leibniz scholarship. Scholars—and even me—are free to use and publish as many Leibniz images as they like as long as the library is credited and the image signatures are included. That's very generous.

I tried to make this project so that AIs would not slurp down the Leibniz images.

Final thoughts

Can you imagine Leibniz with a modern computer?

They still hold great promise for those who, like Leibniz, see them as aides that use language—not a *universal* language, in Leibniz's sense—but in *many* languages. Computers for the betterment of humanity. Not by imposing *control* over humanity, but by acting as *aides* to us, helping us solve our problems. Not by proposing dangerously misinformed mistakes. And they musn't be coercive surveillance units for corporations and/or governments. But of course they currently are.

Did Leibniz anticipate this? What were his politics? Why was he out of favour with even George II, his boss, when Leibniz died—only one person attended his funeral at an unmarked (for years) grave? Was it the conflict with Newton? It had taken its toll on Leibniz's reputation. We now understand Leibniz contributed to calculus more deeply than Newton. In Leibniz's final years, that wasn't known very well, because so much of Leibniz was not widely available, and Newton's forces had waged reputational warfare against Leibniz.

Now there is a library in Hanover, where Leibniz lived and worked for the last 40 years of his life, called the Gottfried Wilhelm Leibniz Library. They have an archive of over 200,000 pages by or owned by Leibniz.

They have already succeeded in making a great deal of it spectacularly online. The online, digital archives are endlessly fascinating. We read Leibniz as *never before*: in his own writing, and across the world. People around the world can now read Leibniz in a way that, previously, was limited to those who could visit the library in person.

The online Leibniz archive will result in much more active reading of Leibniz around the world, and of use of the images in media of one sort or another.

Letters From Leibniz uses 238 archival images of Leibniz's mostly hand-written pages. I'd like to thank the library for making these images available not only for people to see, but freely available, as long as you credit them properly, for use in projects like *Letters From Leibniz*.

Part of the art of a project like *Letters From Leibniz* is in obscuring and revealing. Obscuring what you don't want the focus on. Revealing the focus. Revealing interesting art. Revealing Leibniz.

Letters From Leibniz is a visual reading of Leibniz's visual writing. We see the work of the 17th century genius up close as never before. It's an art project in contemporary visual writing and a study of Leibniz's hand-written, drawn or drafted and only sometimes published work. Letters From Leibniz also involves computer art. It poses riddles by hiding/revealing parts of Leibniz's work and by juxtaposing multiple images. These are digital collages produced with software I wrote. You read them like you read a collage or math or writings of an inventor never far from poetry and philosophy. Leibniz was a poet of the infinitesimal as pretty as Georg Cantor was a poet of the infinite. Also, he anticipated--and his work subsequently propagated--the vision of the machine of mechanized reason.

Leibniz was a poet of the infinitesimal and the characteristica universalis. He was all over the infinite. And he wanted to algebraize calculus, linear algebra, and anything else he studied toward the universalization of the language of mathematics. He wanted to algebraize calculus, for instance, to bring it into the same relation with algebra as the conic sections had via Descartes's coordinate systems, ie analytic geometry.

You watch *Letters From Leibniz* for a few minutes. The full running time for the 500-image slidvid is 71 minutes. That's too long for one session. The idea is that if/when people return, it takes them to something they probably haven't seen yet: if you return to *Letters From Leibniz* via leibniz.vispo.com, it will start at a random digital collage of the 500. It rewards multiple visits.

Letters From Leibniz circles among circles and shards of Leibniz's hand-written linear algebra, binary arithmetic, *I Ching* base 2 hexagrams, base 3 hexagrams, hexadecimal, 17th century computers, calculus, combinatoria, geometry, technica und militaria. Hopefully it's a challenging but intriguing look at Leibniz's visual writing and a unique abstract art experience.

And if you want the never-quite-the-same-twice generative art experience, you can visit Aleph Null using either the Ring brush or the Leibniz Shards brush, both of which use the same set of 238 source images of Leibniz himself.

And if you want to look at the unaltered work of Leibniz, you can view *Visual Leibniz*, a separate slideshow of 238 source images. Featuring the source images in their own slideshow, and linking to the library that provided them, is unlike what we experience with current AI tools that are trained on millions of images but don't acknowledge any of the artists or creators or providers of the images.

But, just as the digital collages in *Letters From Leibniz* are much more meaningfully viewed in relation to also viewing the unaltered source images, AIs should allow us to query concerning the images, artists and providers the AI was trained on. That would be different from the plundering corporate model of AI that is so prevalent now.

Leibniz's dream was of a machine that could help us solve the problems of the world via helping us find the truth of things. This seems a long way from what we currently have in AI. They are not particularly good at reasoning or math, for that matter. They are good at finding a probable next letter, next word, next phrase, next sentence, next paragraph, next chapter, etc. based on a reading of billions of documents.

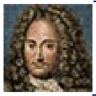
But they have no concept of true and false. If they say something is true, it's only meaningful as something that would probably be said at this point in response to the prompt. They don't even know how to multiply numbers together except as probable outcomes of what they've encountered previously.

So I don't think the man would be especially impressed with what we have in AI now. But he stands for serious development of computing for the betterment of humanity. He was an idealist in the best sense of the word, in that matter.

Let us calculate!

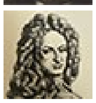
Other Parts of This Project

*Video Intro to Letters From Leibniz* (13 minutes)

 *Letters From Leibniz* (500-image slidvid)

 "Leibniz shards & rings" in *Aleph Null*

 *Visual Leibniz* (slidvid of 238 source images)

 Gottfried Wilhelm Leibniz Library